Lattice gauge theory for graphene

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joint work with V. Mastropietro and M. Porta

GraphITA, May 17, 2011

Outline

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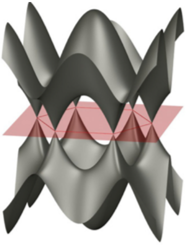
Single-layer graphene

We consider **undoped single-layer graphene**, with no disorder.



The interacting system has strong analogies with **infrared QED**$_{2+1}$ at intermediate coupling strength.

Non-perturbative and unbiased methods are needed to draw even qualitative conclusions.

2D Dirac Fermi gas

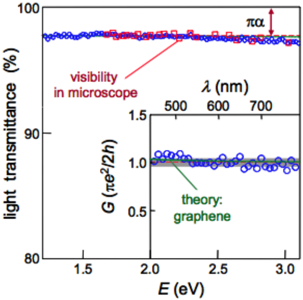
A popular model for undoped clean graphene is **2D Dirac particles in the continuum** with a **static 3D Coulomb** interaction.

Two main drawbacks:

1. Ad hoc regularizations have to be added to cure the spurious UV divergences.

Drawbacks: (1) cutoff dependent results

Sometimes the results are regularization-dependent.
An example: the **universal optical conductivity**.



Nair et al.
Science 2008

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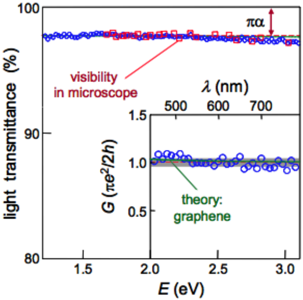

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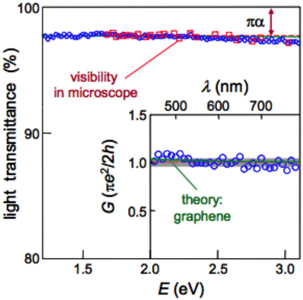

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A big debate on the effect of ad hoc UV cutoffs [Herbut-Juricic-Vafek, Mishchenko, Sheehy-Schmalian, ...]

Theorem [GM 2008 – GMP 2010]. *For the Hubbard model on the honeycomb lattice at* $\mu = 0$ *and* $|U| \leq U_0$,

$$\lim_{\omega\to 0}\lim_{T\to 0}\sigma^T_{xx}(\omega; U) = \frac{\pi}{2}\frac{e^2}{h}$$

Drawbacks: (2) divergent Fermi velocity

2. The static Coulombic interaction is marginal. At 1-loop, RG predicts a **logarithmic growth of the Fermi velocity**, which suggests that retardation effects are important in the understanding of the IR fixed point [Gonzalez-Guinea-Vozmediano].

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Lattice effects are **fully taken into account**:

- no Dirac approximation
- no large-N expansion
- exact lattice Ward Identities

Predictions

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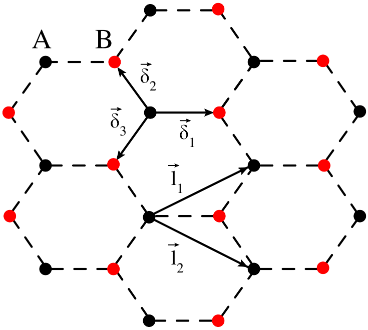
Predictions

1. Line of IR fixed points, parametrized by α. Response functions have anomalous exponents.
2. Lattice gauge invariance guarantees the spontaneous emergence of Lorentz invariance.
3. The Kekulé, CDW and AF responses are enhanced by the interaction: they decay at large distances slower than in the free gas.
4. The K, CDW, AF masses satisfy non-BCS gap eqns, which may admit non-trivial solutions at intermediate coupling.

Outline

The lattice

Let $\Lambda_A = \Lambda$ and $\Lambda_B = \Lambda + \vec{\delta}_i$ be the A- and B-triangular sublattices of the honeycomb lattice.
$a^{\pm}, b^{\pm}$ are creation/annihilation operators on $\Lambda_{A,B}$.



The model

The grandcanonical Hamiltonian is $(\hbar = c = 1)$

$$H_\Lambda = -t \sum_{\substack{\vec{x}\in\Lambda \\ i=1,2,3}} \sum_{\sigma=\uparrow\downarrow} \left(a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_i,\sigma} e^{ie\int_0^1 \vec{A}(\vec{x}+s\vec{\delta}_i)\cdot\vec{\delta}_i\, ds} + c.c. \right)$$

$$+\frac{e^2}{2} \sum_{\vec{x},\vec{y}\in\Lambda_A\cup\Lambda_B} \frac{(n_{\vec{x}}-1)(n_{\vec{y}}-1)}{|\vec{x}-\vec{y}|} + \mathcal{H}_A^{free\ \ field} ,$$

where:

1. $n_{\vec{x}} =$ electronic density at $\vec{x}$;
2. A is a 3D quantized vector potential.

Note: the electron field is coupled nonlinearly to a quantum 3D photon field!

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The two-point function reads:

$$\langle \psi^-_{\mathbf{k},\sigma} \psi^+_{\mathbf{k},\sigma} \rangle = -\frac{1}{Z(\mathbf{k})} \begin{pmatrix} ik_0 & v(\mathbf{k})\Omega^*(\vec{k}) \\ v(\mathbf{k})\Omega(\vec{k}) & ik_0 \end{pmatrix}^{-1} (1+B(\mathbf{k}))$$

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where $B(\mathbf{k})$ is bounded at all orders in e^2 and vanishes at the Fermi points. Anomalous behavior:

$$Z(\mathbf{k}) \sim |\mathbf{k} - \mathbf{p}_F^\pm|^{-\frac{e^2}{12\pi^2}+\cdots},$$

$$1 - v(\mathbf{k}) \sim |\mathbf{k} - \mathbf{p}_F^\pm|^{\frac{2e^2}{5\pi^2}+\cdots}.$$

Local order parameters

Let

$$\zeta^{K}_{\mathbf{x},j} = \sum_{\sigma=\uparrow\downarrow}\left(a^{+}_{\vec{x},\sigma} b^{-}_{\vec{x}+\vec{\delta}_i,\sigma} e^{ie\int_0^1 \vec{A}(\vec{x}+s\vec{\delta}_i)\cdot\vec{\delta}_i\, ds} + c.c.\right)$$

$$\zeta^{CDW}_{\mathbf{x},j} = \sum_{\sigma=\uparrow\downarrow}\left(a^{+}_{\mathbf{x},\sigma} a^{-}_{\mathbf{x},\sigma} - b^{+}_{\mathbf{x}+\boldsymbol{\delta}_j,\sigma} b^{-}_{\mathbf{x}+\boldsymbol{\delta}_j,\sigma}\right)$$

$$\vec{\zeta}^{AF}_{\mathbf{x},j} = a^{+}_{\mathbf{x},\cdot}\tau_3 a^{-}_{\mathbf{x},\cdot} - b^{+}_{\mathbf{x}+\boldsymbol{\delta}_j,\cdot}\tau_3 b^{-}_{\mathbf{x}+\boldsymbol{\delta}_j,\cdot}$$

be the K, CDW, AF local order parameters. Then...

Response functions

$$\langle \zeta^{K}_{\mathbf{x},j}; \zeta^{K}_{\mathbf{y},j'} \rangle \sim \frac{\cos\left(\vec{p}_F^{+}(\vec{x} - \vec{y} + \vec{\delta}_j - \vec{\delta}_{j'})\right)}{|\mathbf{x}-\mathbf{y}|^{4 - \frac{4}{3\pi^2}e^2 + \cdots}}$$

$$\langle \zeta^{CDW}_{\mathbf{x},j}; \zeta^{CDW}_{\mathbf{y},j'} \rangle \sim \frac{1}{|\mathbf{x}-\mathbf{y}|^{4 - \frac{4}{3\pi^2}e^2 + \cdots}}$$

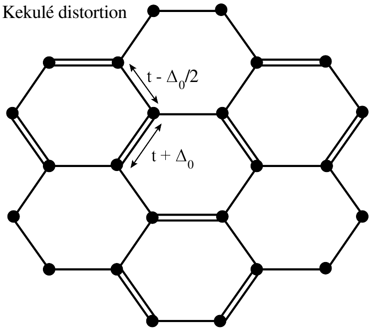
$$\langle \zeta^{AF}_{\mathbf{x},j}; \zeta^{AF}_{\mathbf{y},j'} \rangle \sim \frac{1}{|\mathbf{x}-\mathbf{y}|^{4 - \frac{4}{3\pi^2}e^2 + \cdots}}$$

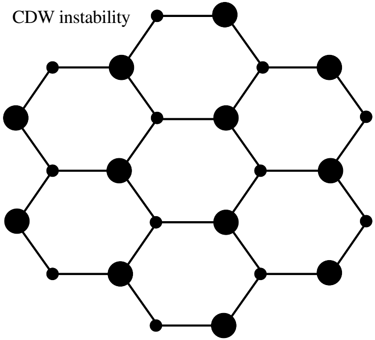
The responses to all the other fermionic bilinears (e.g., Cooper pairs, FM order, lattice current, ...) have faster decay. This suggests that K, CDW, AF are dominant instabilities at intermediate coupling.

Masses

Small Kekulé, charge density wave or Néel modulations are amplified by the interaction:

$$\|\langle \psi^-_{\mathbf{k}'+\mathbf{p}_F^\pm} \psi^-_{\mathbf{k}'+\mathbf{p}_F^\pm} \rangle\| \sim \frac{1}{\sqrt{|\mathbf{k}'|^2+\Delta^2}}, \qquad \Delta = \Delta_0^{1-\frac{2}{3\pi^2}e^2+\cdots}$$





Peierls-Kekulé instability

Under small distortions of the honeycomb lattice

$$t \to t_{\vec{x},j} = t + \phi_{\vec{x},j} \ , \quad \text{with} \quad \phi_{\vec{x},j} = g(\ell_{\vec{x},j} - \bar{\ell}) \ .$$

In the Born-Oppenheimer approximation, the phonon field $\phi_{\vec{x},j}$ is fixed by the variational principle:

$$E_{BOA} = \inf_{\phi} \Big\{ \underbrace{\frac{\kappa}{2g^2} \sum_{\vec{x},j} \phi^2_{\vec{x},j}}_{\textit{elastic energy of the distortion}} \quad + \quad \underbrace{E_0(\{\phi_{\vec{x},j}\})}_{\textit{electronic g.s.e. of the model with a fixed distortion}} \Big\}$$

Extremality condition

The extremality condition for the energy is

$$\kappa\phi_{\vec{x},j} = g^2 \langle \zeta^K_{\mathbf{x},j} \rangle^{\phi}$$

We find that, for any $j_0 \in \{1,2,3\}$,

$$\phi^*_{\vec{x},j} = \phi_0 + \Delta_0 \cos\left(\vec{p}_F^{\,+}(\vec{\delta}_j - \vec{\delta}_{j_0} - \vec{x})\right)$$

is an extremal point of the total energy, provided that $\phi_0 = c_0 g^2/\kappa + \cdots$ for a suitable constant c_0 and that Δ_0 satisfies a non-BCS gap equation.

Non-BCS gap equation

The non-BCS gap equation reads

$$\Delta_0 = \frac{g^2}{\kappa} \int d\mathbf{k}' \frac{Z^{-1}(\mathbf{k}')\Delta(\mathbf{k}')|\Omega(\vec{k}')|^2}{k_0^2 + v^2(\mathbf{k}')|\Omega(\vec{k}' + \vec{p}_F^{+})|^2 + |\Delta(\mathbf{k}')|^2} ,$$

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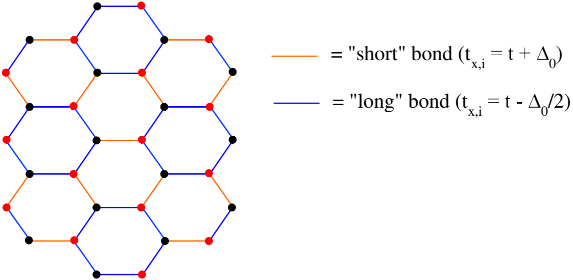
with $\Delta(\mathbf{k}') = \Delta_0 |\mathbf{k}'|^{-\eta_K}$. If Δ_0 is a non-trivial soln, then the system develops a Kekulé pattern.



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Small interactions $\rightarrow g_c = O(\sqrt{t})$.

E.m. interactions lower g_c.

$$g_c \rightarrow 0 \quad when \quad -\frac{7}{12\pi^2} e^2 + \cdots \rightarrow -1$$

The effects of the electron-phonon coupling are dramatically amplified by the e.m. interactions!

Outline

Method of proof - Vanishing beta function

- The proof is based on exact RG methods

$$\mathrm{Tr}\{e^{-\beta H_\Lambda}\} = \int P(d\psi^{(\leq h)}) P(dA^{(\leq h)}) e^{-\mathcal{V}^{(h)}(A^{(\leq h)},\psi^{(\leq h)})}$$

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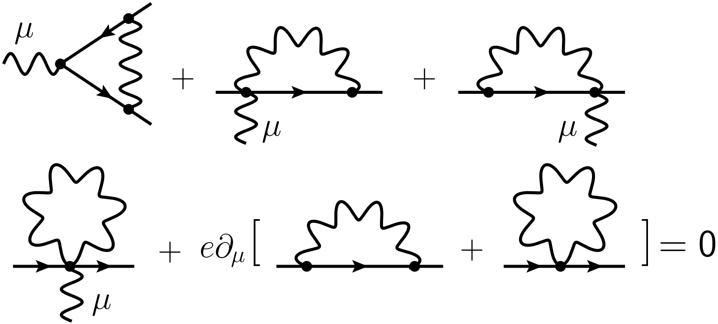
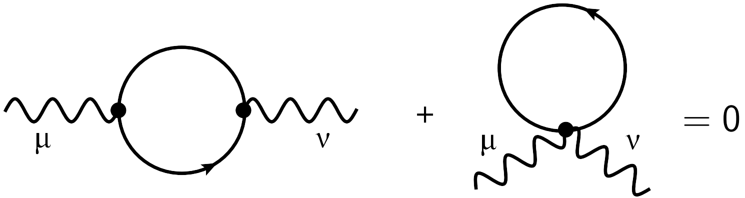
- The vertex $ie_h \int A\psi^+\psi^-$ is marginal. Exact lattice WI imply that the β-function for e_h is 0:



Vanishing photon mass

- The dressed mass of the photon is zero (no screening), again by an exact lattice WI.



- Lattice WIs are ultimately the reason why Lorentz symmetry is dynamically restored.

Limitations and relevance of our RG approach

Our analysis assumes that the effective coupling strength $\alpha = \frac{e^2}{\varepsilon\hbar c}$ is small compared to $\frac{v}{c}$, a condition that is not satisfied by the bare constants. However, our results can be a posteriori extrapolated to larger values of α; moreover, there is no compelling evidence that the effective coupling (after the integration of the “ultraviolet” scales - an intrinsically non-perturbative problem) is large.

Open problems

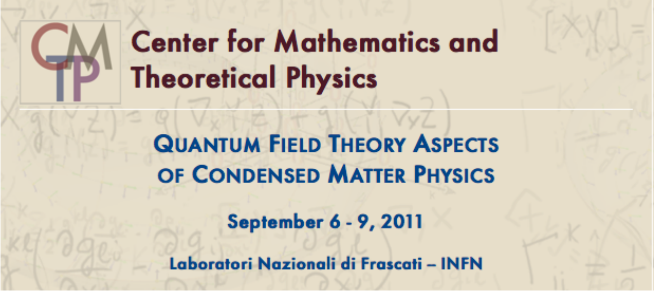
Much remains to be done!

Our methods seem suitable to understand the effects of e.m. interactions on the conductivity, to include the dynamics of a phonon field, bilayer graphene, possibly in a fully non-perturbative fashion.

References

- A.G., V.Mastropietro: *Rigorous construction of ground state correlations in graphene*,PRB2009.
- A.G., V.Mastropietro: *The 2D Hubbard model on the honeycomb lattice*, CMP 2010.
- A.G., V.Mastropietro, M.Porta: *Anomalous behavior in an effective model of graphene with Coulomb interactions*, AHP 2010.
- A.G., V.Mastropietro, M.Porta: *Lattice gauge theory model for graphene*, PRB 2010.
- A.G., V.Mastropietro, M.Porta: *Absence of interaction corrections in the optical conductivity of graphene*, PRB 2011.

CMTP graphene's conference



Organizers

M. Lombardo
V. Mastropietro
M. Vozmediano

Invited speakers

G. Benfatto	G. Jona Lasinio
C. di Castro	M. Katsnelson
V. Falko	E. Lieb
F. Guinea	F. Moraes
D. Haldane	C. Mudry
I. Herbut	J. Pachos
R. Jackiw	G. Volovik