RITUS METHOD AND SUSY-QM: THEORETICAL FRAMEWORKS TO STUDY THE ELECTROMAGNETIC INTERACTIONS IN GRAPHENE

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Graphene is a novel two dimensional material that has opened a new bridge of common interests between the condensed matter and high energy physics communities. It was shown that the low-energy effective theory of graphene in a tight-binding approach is the theory of two species of massless Dirac electrons in a (2+1)-dimensional Minkowski spacetime, each one described by an irreducible representation of the Clifford algebra. Then, the graphene can be studied through the Dirac Hamiltonian in (2+1) dimensions in the zero mass limit. The unique electronic properties of graphene make it a promising novel material to be used in carbon-based electronic devices. It has been considered to be used in electronics and spintronics applications [1, 2, 3]. There are many problems relating electrons in non-uniform magnetic fields of relevance in graphene. In particular, it has been established the possibility to confine quasiparticles in magnetic barriers [4, 5]. This could be feasible creating spatially inhomogeneous, but constant in time, magnetic fields depositing ferromagnetic layers over the substrate of a graphene sample layer [6].

In this work we study the electron propagator in (2+1) dimensions in the presence of external electromagnetic fields in a quantum electrodynamics (QED) framework. In graphene, the massless limit of our findings is of direct relevance. We present the Ritus formalism [7, 8, 9] to obtain the electron propagator in the general case, assuming a magnetic field alone pointing perpendicularly to the plane of motion of the electrons and we explicitly work two time independent examples: (i) the canonical case of a uniform magnetic field and (ii) the case of a non-uniform magnetic field which decays exponentially along the x-axis.

The Ritus method consists in the diagonalization of the electron propagator in external electromagnetic fields in the basis of the operator $(\gamma \cdot \Pi)^2$ with $\Pi^\mu = p^\mu - eA^\mu$. For some class of external static electromagnetic fields, the corresponding Dirac equations can be analyzed within the formalism of supersymmetric quantum mechanics (SUSY-QM) [10, 11], leading us to an exactly solvable model. Under the SUSY-QM framework, the Dirac equation for any external static magnetic field reduces to a corresponding Pauli equation with effective mass $m = 1/2$ and gyromagnetic ratio $g = 2$. Also, there is an important property of SUSY-QM that relates the spectrum and eigenfunctions of the resulting effective hamiltonians and due to the fact that there are two irreducible representations for the Dirac matrices in the (2+1)-dimensional case, for the graphene Hamiltonian there is also a direct relation between the solutions of the electron wave functions with different pseudo-spin eigenvalues.

Exploiting these two formalisms, we also derive the exact Foldy-Wouthuysen (FW) transformation [12] for Dirac fermions in a time independent external electromagnetic field, where the transformation acquires a free form involving the dynamical quantum numbers induced by the field. The FW transformation has been the favourite way to obtain the nonrelativistic limit of the Dirac equation, because it provides a block diagonal form representation of quantum operators and hence of the Dirac Hamiltonian itself. It has been widely used in different stationary metrics in both, gravitational and electromagnetic backgrounds. Finally, going further in the applications of the Ritus formalism in graphene, we also explore the Cini-Touschek transformation [13] which corresponds to the ultrarelativistic limit of the Dirac equation and thus leading to the equation that describes the electrons in graphene.
We expect that apart of its intrinsic theoretical interest, the results presented here could be of usefulness to study the phenomena of confinement in graphene using different configurations of external magnetic fields.

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